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J. Kallinderis
Associate Editor

Limit Cycle Measurements from a Cantilever Beam Attached to a Rotating Body

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I. Introduction

THE simple model of a cantilever beam attached to a rigid, rotating body has been studied extensively inasmuch as it captures the characteristic dynamics of flexible rotating mechanical systems such as helicopter blades, robot arms, and satellite appendages. Some studies have focused on nonlinear dynamic response. The results of one such study,[1] which examined the internally resonant response under periodic, near-resonant excitation, identified distinct classes of periodic responses. Saddle node, pitchfork, and Hopf bifurcation points, associated with the existence and exchange of stability of periodic solutions, were located in the system parameter space. Additionally, the Hopf bifurcation points also indicate regions where stable limit cycle responses characterized as an amplitude modulated time series may exist.[2]

This Note describes the measurement of a stable limit cycle response in an experimental study of the dynamics of a cantilever beam attached to a rotating body. The limit cycle is an amplitude modulated response that would appear near Hopf bifurcations predicted in the theoretical study. A periodic response is presented first. Then as a single-system parameter (the excitation frequency) is varied, the periodic response loses stability and gives way to a stable limit cycle. Such behavior is characteristic of limit cycle (amplitude modulated motion on a two torus) responses measured in other mechanical systems.

II. Experimental Study

A. Apparatus

A schematic diagram of the apparatus is shown in Fig. 1 and consists of the following: a) vacuum vessel, aluminum tank that is evacuated to eliminate aerodynamic loads; b) flexible beam, Lexan, length = 0.52 m, Young's modulus = 2375 MPa, density = 1200 kg/m^3; c) rigid body, solid, aluminum hub, radius $R = 0.076$ m; mounted to a steel drive shaft, the beam is bolted to the rigid body creating a fixed boundary condition; d) four accelerometers, Endevco 7250 A/AM-10 (mass = 1.8 g each), mounted at two locations along the length of the beam with each pair mounted normal to motion in the flapping (out of the plane of rotation) or lead/lag (in the plane of rotation) directions; e) vibration exciter, generic audio speaker mounted vertically to excite motion in the flapping direction only; f) load cell, Wilcoxon Research 5969, mounted on the beam and connected to the exciter by a short, stiff stinger; g) slip ring, 36 channels allowing wire-free connection for signals from the accelerometers, exciter, and load cell; h) electric motor, Parker Hannifin OEM650 Microstep Drive with OEM83-135-MO step motor run at 25,000 steps/revolution; i) motor controller, Galil DMC-1010 motion controller card in personal computer; j) data acquisition and analysis computer; and k) signal generator/analyzer, HP3566A.

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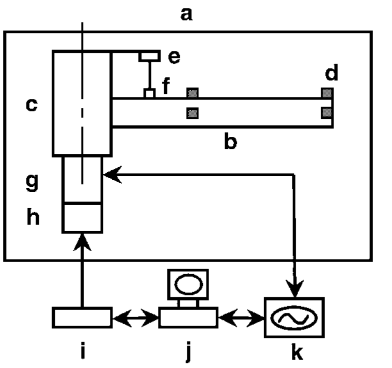


Fig. 1 Schematic of experimental apparatus.

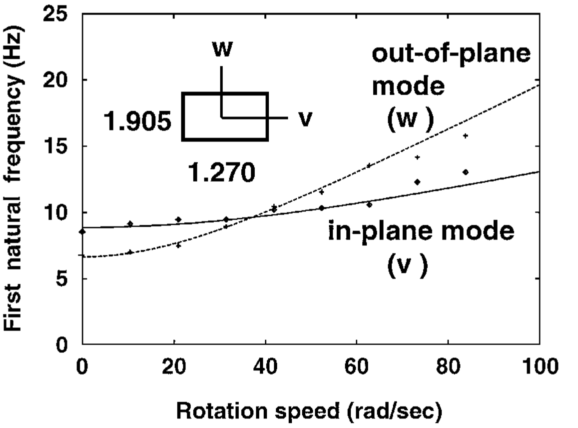


Fig. 2 First natural frequency (theoretical/experimental) of the first in-plane (——/♦) and out-of-plane (- - -/+) bending modes for a beam with a rectangular cross section. The natural frequencies are equal when the rotation speed is 36.9 rad/s.

B. Measurement Conditions

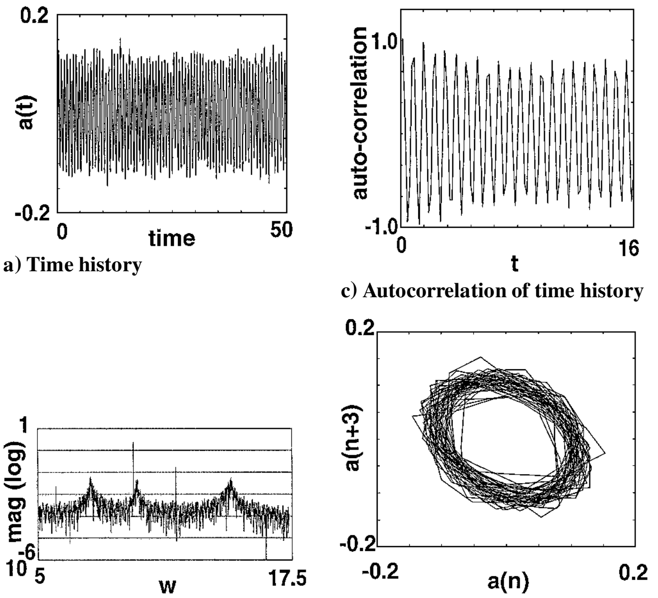
A beam with a rectangular cross section is used. At rest, the natural frequencies in the lead/lag and flapping directions of the beam are not equal. For the class of beams in which the moment of inertia is greater in the lead/lag direction than in the flapping direction, the natural frequencies of lead/lag modes are greater than those of corresponding flapping modes. As the beam rotates, the centrifugal force stiffens the beam in the lead/lag and flapping directions at different rates. As a result for this class of beams, there exists a speed at which the natural frequencies of a lead/lag and a flapping mode are equal.

As shown in Fig. 2, for a beam with a cross section of 1.270×1.905 cm the first natural frequencies are commensurable in a 1:1 ratio at an angular speed of 36.9 rad/s (353 rpm). In Fig 2, the solid and dashed lines represent theoretical predictions, whereas the diamonds and crosses represent measurements taken from an experimental modal analysis survey.[3] Two pairs of accelerometers are attached to the beam at the tip and at an arbitrary midpoint. Time histories from the accelerometers are recorded using the signal analyzer. The analyzer also provides a specified harmonic excitation to the vibration exciter.

C. Sample Results

Two examples are presented here to illustrate the measurement of a typical periodic response and its transition into a limit cycle response. In these examples, the hub rotation speed is a constant 39.94 rad/s (≈350 rpm). In the first example, the excitation frequency is 9.6875 Hz. Figure 3a shows a portion of the time history recorded from the accelerometer near the free end of the beam in the flapping direction. The time history in the lead/lag direction (not shown) has a similar appearance.

Figure 3b shows a fast Fourier transform (FFT) spectrum of the time history. The frequency peak with the greatest amplitude occurs at 9.6875 Hz, which is the excitation frequency. The peak immediately to the right occurs at 9.87 Hz and corresponds to the natural frequency of the flapping mode. The other peaks correspond to the vacuum tank and its harmonics. From the modal survey, the natural



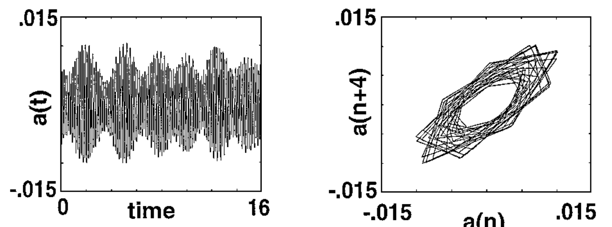
a) Time history

b) FFT spectra of time history

c) Autocorrelation of time history

d) Pseudophase portrait of time history

Fig. 3 Periodic response measured from the accelerometer at the tip of the beam in the out-of-plane direction with excitation frequency 9.6875 Hz.



a) Time history

b) Pseudophase portrait of time history

Fig. 4 Amplitude modulated response measured from the accelerometer at the same location as in Fig. 3, with excitation frequency 9.8 Hz.

frequency of the lead/lag mode was found to be 9.82 Hz. Figure 3c is a portion of the autocorrelation of the time history. It shows that noise is not appreciable. The minor loss of correlation indicates that the motion is nearly periodic. One-quarter of the natural orbital period is chosen to get an open pseudophase portrait. Figure 3d is the pseudophase portrait created following the time delay embedding procedure outlined by Broomhead and King.[4] An embedding delay of three was used for this case.

The excitation frequency was then increased to 9.90 Hz. The excitation amplitude was held constant by monitoring the output from the load cell and manually adjusting the input voltage to the exciter. Figure 4a is a time history again recorded from the accelerometer near the free end of the beam in the flapping direction. It clearly shows a beatinglike appearance typical of an amplitude modulated response. The time history in the lead/lag direction (not shown) has a similar appearance. A pseudophase portrait of the response is shown in Fig. 4b using an embedding delay of four. In this case the thickness of the ellipse arises more from the amplitude modulation of the response than from the noise. The amplitude and period of modulation can be approximated by the motion of a point on a torus.[5] A Poincaré section of the response would be more definitive in determining the aperiodicity of the response. However, one could not be constructed in this case because the data were not sampled at a suitable frequency such as an integer multiple of the excitation frequency.

III. Conclusion

A periodic and a limit cycle response of a cantilever beam attached to a rotating body subject to a harmonic excitation has been presented. The experimental measurement of these motions supports a theoretical study that predicts a variety of nonlinear dynamic responses.

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Application of Parabolized Stability Equations to the Prediction of Jet Instabilities

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Introduction

CLASSICAL inviscid and quasiparallel linear stability theory (LST) cannot account for the natural divergence of jet flow. Because of an accumulation effect, incomplete upstream information could lead to inaccurate predictions downstream. Crighton and Gaster[1] and Plaschko[2] accounted for the diverging effect by the multiple-scales method; however, this is subject to inherent numerical difficulties when neglecting viscosity. Furthermore, neither method can account for the possible distortion of instability waveforms near the jet exit, which could largely influence downstream instabilities. A newly developed method called *parabolized stability equations* (PSE)[3–6] has been used to analyze the streamwise evolution of instability waves in flows that are highly unidirectional, such as boundary layers and free shear layers. That the PSE analysis is able to fully account for the nature of diverging mean flow and to resolve the evolving instabilities without numerical difficulties as seen in LST has made PSE an alternative tool in dealing with jet instabilities at relatively low computational cost compared with schemes based on direct numerical simulation or large eddy simulation.

PSE for Axisymmetric Flow

Yen and Messersmith[7] provide details of the PSE derivation in cylindrical coordinates for a viscous incompressible circular free jet flow. The mean pressure gradients in the axial and radial directions have been discarded because a free jet is considered. For a diverging mean flow, the solution in terms of LST is not strictly justified because the coefficients of the linearized stability equations depend weakly on the axial location z. Both the amplitude function and wave number exhibit streamwise variation due to the

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